ELECTRICAL PROSPECTING METHODS

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**Definitions:**

**Geophysics** is the application of physics to study of the solid earth. It occupies an important position in earth sciences.

**Geophysics** developed from the disciplines of physics and geology and has no sharp boundaries that distinguish it from either.

The use of physics to study the interior of the Earth, from land surface to the inner core is known as **solid earth Geophysics**

Solid Earth Geophysics can be subdivided into **Global Geophysics** or **pure Geophysics** and **Applied Geophysics**.

**Global Geophysics** is the study of the whole or substantial parts of the planet.

**Applied Geophysics** is the study of the Earth's crust and near surface to achieve an economic aim.
**Applied Geophysics**

Comprises the following subjects:

1. Determination of the thickness of the crust (which is important in hydrocarbon exploration).
2. Study of shallow structures for engineering site investigations.
3. Exploration for ground water and for minerals and other economic resources.
4. Trying to locate narrow mine shafts or other forms of buried cavities.
5. The mapping of archaeological remains.
Solid Earth Geophysics

Global or pure Geophysics

Applied Geophysics

Hydro-Geophysics
( Geophysics in Water investigation)

Mining Geophysics
( geophysics for mineral Exploration)

Engineering Geophysics

Exploration Geophysics

Environmental Geophysics
(geophysics in glaciology)

Glacio-geophysics

Archaeo-Geophysics
(in archaeology)
**GEOLOGY**

It involves the study of the earth by direct observations on rocks either from surface exposures or from boreholes and the deduction of its structures, composition and historical evolution by analysis of such observations.

**GEOPHYSICS**

It involves the study of the inaccessible earth by means of physical measurements, usually on or above the ground surface.

**PHYSICAL PROPERTIES OF ROCKS**

* The physical properties of rocks that are most commonly utilized in geophysical investigations are:
  - Density
  - Magnetic susceptibility
  - Elasticity
  - Electrical resistively or conductivity
  - Radioactivity
  - Thermal conductivity

* These properties have been used to devise geophysical methods, which are:
  - Gravity method
  - Magnetic method
  - Seismic method
- Electrical and electromagnetic methods
- Radiometric method
- Geothermal method
ELECTRICAL METHODS

INTRODUCTION

- Electrical prospecting involves the detection of subsurface effects produced by electric current flow in the ground.

- Using electrical methods, one may measure potentials, currents, and electromagnetic fields which occur naturally or are introduced artificially in the earth.

- Electrical methods are often classified, by the type of energy source involved, into Natural or Artificial.

- Natural electrical methods such as self potential (SP), telluric current, magnetotelluric and audio-frequency magnetic fields (AFMAG), depend on naturally occurring fields and in this respect resemble gravity and magnetic prospecting.

- Artificial electrical methods such as resistivity, equipotential point and line, mise-a-la-masse, electromagnetic (EM) and induced polarization (IP) are similar to seismic methods.

- Only one electrical method (i.e. telluric method) can penetrate to the depths where oil and gas are normally found.

- Electrical methods are more frequently used in searching for metals, groundwater, archaeology, and engineering problems because most of them have proved effective only for shallow exploration, seldom giving information on subsurface features deeper than 1000 or 1500 ft.
TELLURIC CURRENT METHOD

- Telluric currents are natural electric currents that flow in the earth crust in the form of large sheets.

- Their presence is detected easily by placing two electrodes in the ground separated by a distance of about 300 meters or more and measuring the potential difference between them.

- The origin of these telluric currents is believed to be in the ionosphere and is related to the continuous flow of charged particles from the sun which becomes trapped by the lines of force of the earth's magnetic field.

- If the ground in a given area is horizontally stratified and the surface of the basement rocks is also horizontal, at any given moment the density of the telluric current is uniform over the entire area.

- In the presence of geologic structures such as anticlines, synclines and faults, the distribution of current density is not uniform over the area.

Fig. 1: Flow of telluric current over an anticline. Ellipse and circles indicates telluric field intensity.
As a function of direction with respect to axis of anticline

- The current density is a vector quantity and the vector is larger when the telluric current flows at right angles to the axis of an anticline than when the current flows parallel to the axis.

- By plotting these vectors we obtain ellipse over anticlines and synclines and circle where the basement rocks are horizontal.

- The longer axis of the ellipse is oriented at right angles to the axis of the geologic structure.

**MAGNETO-TELLURIC METHOD**

- It is similar to the telluric current method but has the advantage of providing an estimate of the true resistivity of the layer.

- Measurements of amplitude variations in the telluric field ($E_x$) and the associated magnetic field ($H_y$) determine earth resistivity.

- Magnetotelluric measurements at several frequencies provide information on the variation of resistivity with depth because the depth of penetration of EM waves is a function of frequency.

- The method is useful in exploration to greater depths (i.e. petroleum exploration in Russia).
ELECTRICAL PROPERTIES OF EARTH'S MATERIALS

- Several electrical electrical properties of rocks and minerals are significant in electrical prospecting. These are:
  1. electrical potentials.
  2. Electrical conductivity (or the inverse electrical resistivity)
  3. Dielectric constant.

- The electrical conductivity is the most important while the others are of minor significance.

- The electrical properties of most rocks in the upper part of the earth's crust are dependent primarily on the amount of water in the rock and its salinity.

- Saturated rocks have high conductivities than unsaturated and dry rocks.

- The higher the porosity of the saturated rocks, the higher its conductivity.

- The conductivity of rocks increases as the salinity of saturating fluid increases.

- The presence of clays and conductive minerals also increases the conductivity of the rocks.

- The electrical conductivity of Earth materials can be studied by two ways:
- Measuring the electrical potential distribution produced at the Earth's surface by an electric current that is passed through the earth.

- Detecting the electromagnetic field produced by an alternating electric current that is introduced into the earth.

**DIRECT CURRENT (D.C.) RESISTIVITY METHOD**

- The most common used methods for measuring earth resistivity are those in which current is driven through the ground using galvanic contact.

- Generally, four – terminal electrode arrays are used since the effect of material near the current contacts can be minimized.

- Current is driven through one pair of electrodes (A & B) and the potential established in the earth by this current is measured with the second pair of electrodes (M & N) connected to a sensitive voltmeter. It is then possible to determine an effective or apparent resistivity of the subsurface.

![Diagram of current flow through earth](image)

Fig.2: Current flow through earth
- Anomalous conditions or in-homogeneities within the ground, such as electrically better or poorer conducting layers are inferred from the fact that they deflect the current and distort the normal potentials.

- In studying the variation of resistivity with depth, as in the case of a layered medium, the spacing between the various electrodes are generally increased with larger spacing, the effect of the material at depth on the measurements becomes more pronounced. This type of measurements is called a **vertical sounding** or **electrical coring**.

- In studying lateral variations such as might be associated with dike like structures or faults, a fixed separation is maintained between the various electrodes and the array is moved as a whole along a traverse line. This type of measurement is called **horizontal profiling** or **electrical trenching**.

- The chief drawback of the resistivity method is the practical difficulty involved in dragging several electrodes and long wires over rough wooded or rocky terrain. This made the EM method more popular than resistivity in mineral exploration.

- In the 1920 the technique of the method was perfected by Conrrad Schlumberger who conducted the first experiments in the field.

- In practice, there are other complicated electrical effects which may create potentials other than that caused by simple ohmic conduction of the applied current. For example:

  1) Electrical potentials can be developed in the earth by electrochemical actions between minerals and the
solutions with which they are in contact. No external currents are needed in this case. The detection of these potentials forms the basis of the **self potential (SP)** method of exploration for ore bodies such as pyrite.

2) Electrical charges sometimes accumulate on the interfaces between certain minerals as a result of the flow of electric current from an external source. The method of **Induced Polarization (IP)** is based on this phenomenon in the search for disseminated ores and clay minerals.

3) Slowly varying potentials are caused by natural (telluric) current flowing inside the earth by the ionospheric currents. They are capable of extending deep into the earth's crust.

- The resistivity method provides a quantitative measure of the conducting properties of the subsurface. This technique can be used to find the depths of layers in the earth having anomalously high or low conductivities and to determine the depth, approximate shape of ore bodies with anomalous resistivity.

**BASIC PRINCIPLES**

**Electrical Resistivity (the inverse is electrical conductivity)**

- The relative abilities of materials to conduct electricity where a voltage is applied are expressed as **conductivities.**

- Conversely, the resistance offered by a material to current flow is expressed in terms of **resistivity.**
- For almost all electrical geophysical methods, the true or more scientifically, the specific resistivity of the rock is of interest.

- The true resistivity of a rock unit is defined as being equal to the resistance of a unit cube of the rock.

- Consider an electrically uniform cube of side length "L" through which a current (I) is passing.

![Diagram of a homogenous block with applied current and potential drop](image)

**Fig.3:** (A) Basic definition of resistivity across a homogenous Block of side length L with an applied current I and potential drop between opposite faces of V. (B) the electrical circuit equivalent, where R is a resistor

- The material within the cube resists the conduction of electricity through it, resulting in a potential drop (V) between opposite faces.

- It is well known that: The resistance (R) in ohm of a sample is directly proportional to its length (L) of the resistive material and inversely proportional to its cross sectional area (A) that is:

  \[ R \propto \frac{L}{A} \]
\[ R \propto \frac{1}{A} \]
\[ R \propto \frac{L}{A} \]
\[ R = \rho \left( \frac{L}{A} \right) \quad \ldots \quad (1) \]

Where \( \rho \), the constant of proportionality is known as the electrical resistivity, a characteristic of the material which is independent of its shape or size. The constant of proportionality is the "true" resistivity (\( \rho \)).

- According to Ohm's law:

\[ R = \frac{\Delta V}{I} \quad \ldots \quad (2) \]

Where \( \Delta V = V_2 - V_1 \), the potential difference across the resistor and \( I = \) the electric current through it.

"R" is the resistance of the cube

Substituting equation (1) in equation (2) and rearranging we get:

\[ \rho = \left( \frac{\Delta V}{I} \right) \left( \frac{A}{L} \right) \quad \ldots \quad (3) \]

- Equation (3) may be used to determine the resistivity (\( \rho \)) of homogeneous and isotropic materials in the form of regular geometrical shapes such as cylinders, cubes, ….

- In a semi-infinite material, the resistivity at every point must be defined using Ohm's law which states that the electrical field strength (\( E \)) at a point in a material is proportional to the current density (\( J \)) passing that point:

\[ E \propto J \]
\[ E = \rho J \]
\[ \rho = \frac{E}{J} \quad \text{(Ohm's law)} \]

(E) expressed in volts/meters
(J) expressed in amperes/meter\(^2\)

- The unit or resistivity in the mks system (meter-kilogram-second) is ohm-meter (\(\Omega\ m\)) which is convenient for expressing the resistivity of earth materials.

- Ohm-centimeter can also be used where:
  
  \[ 1 \text{ ohm-m} = 100 \text{ ohm-cm} \]

- Ohm' law: for an electrical circuit, Ohm's law gives \( R = \frac{V}{I} \), where (V) and (I) are the potential difference across a resistor and the current passing through it, respectively. This can be written alternatively in terms of the electric field strength (E) and current density (J) as:

\[ \rho = \frac{E}{J} \]
\[ \rho = \frac{(VA)}{(IL)} \]

- The inverse of resistivity (1/\(\rho\)) is conductivity (\(\sigma\)) which has units of siemens/meter (S/m) which are equivalent to mhos/meter (\(\Omega^{-1}\text{m}^{-1}\)).

- **The potential in a homogeneous medium**

  **A. One current electrode at surface** (point source of current).
• Let a current of a strength (I) enters at point $C_1$ on the ground surface.

• This current will flow radially from the point of entry and at any instant its distribution will be uniform over a hemispherical surface of the underground of resistivity ($\rho$).

• At point (P), a distance (R) away from the source the potential is given by:

$$V = \frac{I\rho}{2\pi R}$$

$$V_P = \left(\frac{I\rho}{2\pi}\right) \left(\frac{1}{R}\right)$$

**B. Two current electrodes at surface**

- Diagram showing current flow and potential distribution.
In practice we have two electrodes, one positive (A), sending current into the ground and the other negative (B), collecting the returning current.

The potential at any point "P" in the ground will then be:

\[ V = \rho I / 2\pi \left(1/R_1 - 1/R_2\right) \]

When two current electrodes, A & B are used and the potential difference, \( \Delta V \), is measured between two measuring electrodes M and N, we get:

\[ V_{A,M} = \rho I/2\pi \left(1/AM\right) \]  
\[ \text{...... potential at M due to positive} \]
\[ \text{electrode A.} \]

\[ V_{A,N} = \rho I/2\pi \left(1/AN\right) \]  
\[ \text{...... potential at N due to positive} \]
\[ \text{electrode A.} \]

\[ V_{B,M} = \rho I/2\pi \left(1/BN\right) \]  
\[ \text{...... potential at N due to negative} \]
\[ \text{electrode B.} \]

\[ V_{B,N} = \rho I/2\pi \left(1/BM\right) \]  
\[ \text{...... potential at M due to negative} \]
\[ \text{electrode B.} \]

\[ V_{M(A,B)} = \rho I/2\pi \left(1/AM - 1/BM\right) \]  
\[ \text{...... Total potential at M} \]
\[ \text{due to A & B} \]

\[ V_{N(A,B)} = \rho I/2\pi \left(1/AN - 1/BN\right) \]  
\[ \text{...... Total potential at N} \]
\[ \text{due to A & B} \]

The net potential difference is:

\[ \Delta V_{MN(A,B)} = V_{M(A,B)} - V_{N(A,B)} \]

\[ \Delta V = \rho I/2\pi \left(1/AM - 1/BM - 1/AN + 1/BN\right) \]

\[ \rho = \Delta V/I \left\{ 2\pi/ \left(1/AM - 1/BM - 1/AN + 1/BN\right) \right\} \]
This equation is a fundamental equation in D.C. electrical prospecting.

The factor \( \frac{2\pi}{(1/AM - 1/BM - 1/AN + 1/BN)} \) is called the geometrical factor of the electrode arrangement and generally designed by letter (K):

\[
\rho = K \left( \frac{\Delta V}{I} \right)
\]

If the measurement of (\( \rho \)) is made over a homogeneous and isotropic material, then the value of (\( \rho \)) computed from the above equation will be the true resistivity.

If the medium is inhomogeneous and (or) anisotropic then the resistivity computed is called an apparent resistivity (\( \rho \)).

The apparent resistivity is the value obtained as the product of a measured resistance \( \mathcal{R} \) and a geometric factor (K) for a given electrode array.

**Effects of Geologic variations on the Resistivity measurements**

1- High resistivity material at depth:
- Lines of current flow tend in general to avoid high resistivity material.

- From the above figure we observe that the current density will be increased in the upper layer.

- If a small electrode spacing is used, a shallower pattern of current flow will be produced as shown in the following figure.

- The \((\rho_2)\) material will have less influence on it.

\[\text{Fig.6}\]

2- Low resistivity material at depth:

\[\text{Fig.7}\]

- Lines of currents flow tend to be attracted toward low resistivity material.

- In the above figure we observe that the current density will be decreased in the upper layer.

- From the above explanation we can conclude that:
a. The variation of true resistivity with increasing depth should appear as a variation of apparent resistivity with increasing electrode spacing.

b. The trend will be parallel: if apparent resistivity is increasing at a particular electrode spacing, then true resistivity will also be increasing at some corresponding depth and vice versa.

c. An abrupt change in resistivity at a particular depth must appear as a smooth and gradual change in the apparent resistivity curve.

d. The effect of a shallow boundary will appear at a smaller electrode separation, whereas the effect of a deeper boundary will appear at larger electrode separations.

e. No simple relationship exist between electrode spacing and depth, since the effect of a boundary appears gradually in the data as the electrode spacing is increased.

3- Effect of topographic relief:

![Diagram](image)

Fig.8

- Since the resistivity of air is very large, the lines of current flow must be strongly deflected to the left.

- This increases the current density throughout the region and the measured apparent resistivity is increased.
The above figure shows resistivity measurements made near a vertical cliff.

We note that the effect becomes larger as the cliff is approached more closely.

4- Potential and current distribution across a boundary

At the boundary between two media of different resistivities, the potential remains continuous while the current lines are refracted according to the law of tangents as they pass through the boundary.

From the figure below, the law of refraction of current lines can be written as:
\[ \rho_1 \tan \alpha_1 = \rho_2 \tan \alpha_2 \]

If \( \rho_2 < \rho_1 \) the current lines will be refracted away from the normal and vice versa.

![Fig.9: Refraction of current lines crossing a boundary between two media of different resistivities](image)

RESISTIVITIES OF ROCKS AND MINERALS

The resistivity (\( \rho \)) of rocks and minerals displays a wide range. For example, graphite has a resistivity of the order
of $10^{-5}$ ohm-m, whereas some dry quartzite rocks have resistivities of more than $10^{12}$ ohm-m.

- No other physical property of naturally occurring rocks or soils displays such as wide range of values.

- The resistivity of the geological materials ranges from $1.6 \times 10^{-8}$ $\Omega$m for native silver to $10^{16}$ $\Omega$m for pure sulphur.

- Igneous rocks tend to have the highest resistivities, sedimentary rocks tend to be most conductive due to their high pore fluid content. Metamorphic rocks have intermediate but overlapping resistivities.

- The age of a rock is an important consideration: a Quaternary volcanic rock may have a resistivity in the range 10-200 $\Omega$m while that of an equivalent rock but Pre-Cambrian in age may be an order of magnitude greater.

- Some minerals such as pyrite, galena and magnetite are commonly poor conductors in massive form yet their individual crystals have high conductivities.

- Hematite and sphalerite, when pure, are virtual insulators, but when combined with impurities they can become very good conductors (with resistivities as low as 0.1 $\Omega$m).

- Resistivities for sandy material are about 100 $\Omega$m and decrease with increasing clay content to about 40 $\Omega$m.

- The objective of most modern electrical resistivity surveys is to obtain the resistivity models for the subsurface because it is these that have geological meaning.
Factors controlling the resistivity of earth materials:

- The electrical current is carried through the earth material by either:

  1) Motion of free electrons or ions in the solid. This is important when dealing with certain kinds of minerals such as graphite, magnetite or pyrite.

  2) Motion of ions in the connate water, come from the dissociation of salts such as sodium chlorite, magnesium chloride. This is important when dealing with engineering and hydrogeology.

- For water bearing rocks and earth materials, the resistivity decreases with increasing:

  1) Fractional volume of the rocks occupied by water (i.e. water content).

  2) Salinity or free ion content of the connate water (i.e. water quality).

  3) Interconnection of the pore spaces (i.e. permeability and porosity).

  4) Temperature.

From the proceeding, we may infer that:

A. Materials which lack pore spaces will show high resistivity such as:
   a) Massive limestone.
   b) Most igneous and metamorphic rocks such as granite and basalt.
B. Materials whose pore spaces lacks water will show high resistivity such as: dry sand or gravel and ice.

C. Materials whose connate water is clean (free from salinity) will show high resistivity such as clean gravel or sand, even if water saturated.

D. Most other materials will show medium or low resistivity, especially if clay is present, such as: clay soil and weathered rocks.

- The presence of clay minerals tends to decrease the resistivity because:
  
a) The clay minerals can combine with water.
  
b) The clay minerals absorb cations in an exchangeable state on the surface.
  
c) The clay minerals tend to ionize and contribute to the supply of free ions.

- As a rough guide, we may divide earth materials into:

  a) Low resistivity  less than 100 $\Omega$m.
  b) Medium resistivity  100 to 1000 $\Omega$m
  c) High resistivity  greater than 1000 $\Omega$m.

---

**EQUIPMENTS FOR RESISTIVITY FIELD WORK**

- The necessary components for making resistivity measurements include:
  
1) Power source
2) Meter for measuring current and voltage (which may be combined in one meter to read resistance)
3) Electrodes
4) Cables
5) Reels.

1) Power Source

- The power may be either D.C. or low frequency A.C., preferably less than 60 Hz.

- If D.C. is used, a set of B- batteries (45 to 90 volts) may be used, connected in series to give several hundred volts total.

- For large scale work it is preferable to use a motor generator having a capacity of several hundred watts.

- To avoid the effect of electrolytic polarization caused by unidirectional current, the d.c. polarity should be reversed periodically with a reversing switch.
Fig. 10: equipment for measuring resistivity
2) Meters

- With d.c. source, the current is measured with a d.c. milli-ammeter, whose range should be from about 5 to 500 mA,
depending on the electrode spacing, type of ground and power used.

- Potential is normally measured with a d.c. voltmeter of high input impedance.

**Potentiometer:**

- The voltage between the measuring electrodes is usually measured with a potentiometer (a voltmeter may be used).

3) **Electrodes**

- **Current Electrodes:**

  - They are generally steel, aluminum or brass. Stainless steel is probably best for combined strength and resistance to corrosion. They are driven a few inches into the ground.

  - In dry ground, the soil around the electrodes may have to be moistened or watered to improve contact.

  - To reduce the contact resistance, many stakes driven into the ground a few feet apart and connected in parallel.

  - Where bare rock is exposed at the surface it may not be possible to drive a stake into the ground, and in such a case a current electrode may be formed by building a small mud puddle around a piece of copper screening.

- **Potential electrodes**

  - Contact resistance is not important in case of potential electrodes as in case of current electrodes.

  - Potential electrodes must be stable electrically. When a copper or steel stakes is driven into the ground, the potential difference between the metal in the electrode and
the electrolytic solution in the soil pores may take minutes to reach equilibrium and may vary erratically during this time.

- A stable electrode may be obtained by using a non-polarizing electrode, an electrode consisting of a metal bar immersed in a solution of one of its salts carried in ceramic cup. Such electrodes are called "porous pots". The metal which is used may be copper and the solution copper sulphate or silver metal in a silver nitrate solution may be used.

- Let the solution carries an excess of salt in crystal form to become saturated and the potential remains constant.

- The ceramic cups used in porous pot electrodes must be permeable enough that water flows slowly through to maintain contact between the electrode and the soil moisture.

4- Cables:

- Cables for connecting the current electrodes to the power source or the measuring (potential) electrodes to the measuring circuit present no special requirements.

- Wire must be insulated and should be as light as possible. Plastic insulation is more durable than rubber against abrasion and moisture.

Instrumental problems:

- The three most important respects are as follows:

1. If the potential measuring circuit draws any current, a polarization voltage may develop at the contact between
the potential electrodes and the soil. This will appear as a spurious voltage in series with the true voltage.

2. If the potential electrodes are metallic, electro-chemical potential may arise due to interaction with soil fluid. This problem can be controlled by the use of non-polarizing potential electrodes.

3. Natural earth currents may be flowing past the electrodes, producing extraneous natural potentials which add to the desired artificial potentials.

**Apparent Resistivity**

- All resistivity techniques in general require the measurement of apparent resistivity.
In making resistivity surveys a direct current or very low frequency current is introduced into the ground via two electrodes (A & B) and the potential difference is measured between a second pair of electrodes (M & N).

If the four electrodes are arranged in any of several possible patterns, the current and potential measurement may be used to calculate apparent resistivity.

If the measurement of \((\rho)\) is made over a semi-infinite space of homogeneous and isotropic material, then the value of \((\rho)\) will be true resistivity of the material.

If the medium is in-homogenous and or anisotropic, the resistivity is called apparent resistivity \((\rho_a)\).

**Electrode configuration and field procedure:**

For field practice a number of different surface configurations are used for the current and potential electrodes.

Both sets of electrodes are laid out along a line for all of those arrangements.

The current electrodes are generally but not always placed on the outside of the potential electrodes.

The value of the apparent resistivity depends on the geometry of the electrode array used, as defined by the geometric factor "K".

There are three main types of electrode configuration.
1) Wenner array

In the Wenner array four electrodes are equally spaced along a straight line so that AM = MN = NB = a.

It was known before that:

\[ \rho = \frac{\Delta V}{I} \left\{ \frac{2\pi}{\frac{1}{AM} - \frac{1}{BM} - \frac{1}{AN} + \frac{1}{BN}} \right\} \]

For this configuration, the apparent resistivity reduces to:

\[ \rho_a = \frac{\Delta V}{I} \left\{ \frac{2\pi}{\frac{1}{a} - \frac{1}{2a} - \frac{1}{2a} + \frac{1}{a}} \right\} \]

\[ \rho = \frac{\Delta V}{I} \left\{ \frac{2\pi}{2/a - 1/a} \right\} \]

\[ \rho = 2\pi.\frac{\Delta V}{I} (a) \]

\[ \rho_a = 2\pi a.\frac{\Delta V}{I} \]

\(2\pi a\) is called geometrical factor.
2) Schlumberger Array

This array is the most widely used in electrical prospecting.

Four electrodes are placed along a straight line on the earth surface as in the Wenner array, but with \( AB \geq 5 \text{ MN} \).

Two closely spaced measuring electrodes (M & N) are placed midway between two current electrodes (A & B).

In lateral exploration with the Schlumberger array, it is permissible to measure potential somewhat off the line between fixed current electrodes.

In addition to the potential associated with current introduced into the earth by the current electrodes, the potential difference as read may include spurious electrochemical potentials between the electrodes and electrolytes in the earth.

Often non-polarizing electrodes (such as copper sulfate porous pots) are used to avoid such effects.
Spurious electrode potentials are frequently canceled by using low frequency alternating current.

In this case and according to the above equation, the configuration factor can be proved as follows:

If \( MN = l \) and \( AB = L \) then

\[
\rho_a = 2\pi \cdot \Delta V/I \left\{ \frac{1}{1} \right\} \left\{ \frac{1}{(L/2-l/2)} - \frac{1}{(L/2+l/2)} - \frac{1}{(L/2+l/2)} \right\}
\]

\[
\rho = 2\pi \cdot \Delta V/I \left\{ \frac{1}{1} \right\} \left\{ \frac{2}{L-l} - \frac{2}{L+l} - \frac{2}{L+l} + \frac{2}{L-l} \right\}
\]

\[
\rho = 2\pi \cdot \Delta V/I \left\{ \frac{1}{4} \right\} \left\{ \frac{(2L-4I)}{(2L+2I)} \right\}
\]

\[
\rho = 2\pi \cdot \Delta V/I \left\{ \frac{1}{4} \right\} \left\{ \frac{(4L+4I-4L+4I)}{(L^2-l^2)} \right\}
\]

\[
\rho = 2\pi \cdot \Delta V/I \left\{ \frac{1}{4} \right\} \left\{ \frac{(8I)}{(L^2-l^2)} \right\}
\]

\[
\rho = \pi \cdot \Delta V/I \left\{ \frac{1}{4} \right\} \left\{ \frac{(L^2-l^2)}{4I} \right\}
\]

\[
\rho_a = \pi \cdot \Delta V/I \left\{ \frac{1}{4} \right\} \left\{ \frac{(AB)^2 - (MN)^2}{MN} \right\}
\]

\[
\rho_a = K \cdot \Delta V/I
\]

K is the configuration factor.

The number (4) in the last equation is removed because:

\( L^2 = (AB)^2 \) or \( 4(AB/2)^2 \) \&

\( I^2 = (MN)^2 \) or \( 4(MN/2)^2 \)

3) Dipole – Dipole array (Double – dipole system)

- In a dipole – dipole array, the distance between the current electrodes A and B (current dipole) and the distance
between the potential electrodes M and N (measuring dipole) are significantly smaller than the distance (r) between the centers of the two dipoles.

- The potential electrodes (M & N) are outside the current electrodes (A & B).

\[ \text{Bipole-Bipole Polar Array} \]

\[ \text{Increase } R, \text{ b "fixed"} \]

\[ \text{Fig 13} \]

- If the separation between both pairs of electrodes is the same (a), the expression for the geometric factor is:

\[ K = \pi \left\{ \left( \frac{r^3}{a^2} \right) - r \right\} \]

- If each pair has a contact mutual separation (a) and (na) is the distance between the two innermost electrodes (B & M), then:

\[ \rho_a = K \cdot \Delta V/I \]

\[ \rho_a = \{ \pi n (n + 1) (n + 2) a \} (\Delta V/I) \]
Various dipole – dipole arrays:

- Several different dipole – dipole configurations have been suggested as shown in this figure:

(Bipole-Bipole Equatorial Array
(Increase R, b "fixed")
When the angle \( \theta \) equals 90°, the azimuth array and the parallel array reduce to the equatorial array.

When the angle \( \theta = 0 \), the parallel and radial arrays reduce to the polar or axial array.

The advantage of dipole–dipole arrays is that the distance between the current source and the potential dipole can be increased almost indefinitely, being subject only to instrumental sensitivity and noise whereas the increase of electrode separation in the Wenner and Schlumberger arrays is limited by cable lengths.
4) Pole – dipole (or pole – bipole or tripole or the half Schlumberger) array:

- When one of the current electrodes, say B, is very far removed from the measurement area, the electrode A is referred to as a current pole.

- Bipole means enlarging the length of the current electrodes

5) Dipole – pole array

- When one of M & N electrodes is far removed, the remaining electrode is referred to as a potential pole.

- If the current dipole AB is then short compared with its distance from the potential pole we have a dipole – pole array.

6) Pole – pole array

- A pole – pole arrangement will be obtained when one of A, B and one of M, N are removed to infinity.
RESISTIVITY METHOD FIELD PROCEDURES

- There are only two basic procedures in resistivity work. The procedure to be used depends on whether we are interesting in lateral or vertical variations in resistivity.

- The first is called horizontal or trenching profiling and the second is called electric drilling or sounding.

a) Vertical Electrical Sounding (VES or drilling)

\[ R_M = \frac{V}{I} \]
\[ \rho = 2 \pi a R_M \]

Objective: Detect and characterize various layers at depth.

Procedure for a 4-electrode DC resistivity "sounding". An expanding array senses to greater depth.
Electrical sounding is the process by which the variation of resistivity with depth below a given point on the ground surface is deduced and it can be correlated with the available geological information in order to infer the depths (or thicknesses) and resistivities of the layers (formations) present.

The procedure is based on the fact that the current penetrates continuously deeper with the increasing separation of the current electrodes.

When the electrode separation, $C_1 C_2$, is small compared with the thickness, $h$, of the upper layer, the apparent resistivity as determined by measuring ($\Delta V$) between the potential electrodes, $P_1 P_2$, would be virtually the same as the resistivity of the upper layer ($\rho_1$).
As the electrode separation is increased a greater fraction of current will penetrate deeper, the lines of current flow being distorted at the boundary.

"Sounding" is A Matter of Scale

Case 1 (Small a-spacing): Samples shallow depth

\[ \rho_a = 2\pi a (V/l) \]

In the above figure, when the current electrode separation (A & B) is small compared with the thickness (h) of the upper layer, the apparent resistivity as determined by measuring (ΔV) between the potential electrodes (M & N) would be the same as the resistivity of the upper layer. This is because a very small fraction of the current would penetrate in the substratum below the boundary.
● At spacings which are very large compared with (h), a greater fraction of current will penetrate deeper and the apparent resistivity approaches (ρ₂) because the fraction of current confined to the surface layer becomes negligible.

● In the case of the dipole-dipole array, increased depth penetration is obtained by increasing the inter-dipole separation, not by lengthening the current electrode array.

● The position of measurement is taken as the mid-point of the electrode array.

● For a depth sounding, measurements of the resistance (ΔV/I) are made at the shortest electrode separation and then at progressively larger spacings.

● At each electrode separation a value of apparent resistivity (ρₐ) is calculated using the measured resistance in conjunction with the geometric factor for the electrode configuration and separation being used.

● The values of apparent resistivity are plotted on a graph (field curve), the x- and y-axes of which represent the logarithmic values of the current electrode half-separation (AB/2) and the apparent resistivity (ρₐ), respectively.

![Fig 17](image-url)
Wenner configuration (Sounding)

- All the four electrodes have to be moved after each measurement so that the array spacing, $a$, is increased by steps, keeping the midpoint of the configuration fixed.

- The apparent resistivity is obtained from the equation of the Wenner array configuration.

- It must be remembered that here, as in all resistivity measurements, $\Delta V$ represents the measured voltage between M and N minus any self potential voltage between M and N observed before the current is passed.

Schlumberger sounding

- $\Delta V$ represents the measured voltage between M and N minus any self potential voltage between M and N observed before the current is passed.
- The potential electrodes (M & N) are kept at a fixed spacing (b) which is no more than 1/5 of the current electrode half spacing (a).

- The current electrodes (A & B) are moved outward symmetrically in steps.

- At some stages the MN voltage will, in general, fall to a very low values, below the reading accuracy of the voltmeter in which case the distance MN is increased (e.g. 5 or 10 fold), maintaining of course, the conditions MN<<AB.

- The measurements are continued and the potential electrode separation increased again as necessary until the VES is completed.

- It is advisable then to have an overlap of two or three readings with the same AB and the new as well as the old MN distance.

- The \( \rho_a \) values with the two MN distances but the same AB distance sometimes differ significantly from each other.

- In this case, if the results are plotted as \( (\rho_a) \) against AB (or AB/2) on a double logarithmic paper, each set of \( (\rho_a) \) values obtained in the overlapping region with one and the same MN will be found to lie on separate curve segments, displaced from each other.

![Fig 20: Displacement of segments in Schlumberger sounding](image)
- The different segments must be suitably merged to obtain a single smoothed sounding curve.

- It is sufficient to shift a segment obtained with a larger MN towards the adjoining previous one obtained with the smaller MN.

**Dipole – Dipole sounding**

![Diagram of dipole-dipole sounding](image)

Fig 21

- The dipole – dipole array is seldom used for vertical sounding as large and powerful electrical generators are required.

- The distance between the two dipoles (i.e. AB & MN) is increased progressively to produce the sounding.

- Once the dipole length has been chosen (i.e. the distance between the two current electrodes and between the two potential electrodes), the distance between the two dipoles is then increased progressively to produce the sounding.

**Presentation of the sounding data**

- After computing apparent resistivity for each reading, the data is plotted as a function of the electrode spacing (AB/2) on double logarithmic paper with the electrode spacing on the abscissa and the apparent resistivity on the
ordinate. The curve obtained is called an electrical sounding curve.

**Advantages of using logarithmic coordinates**

1. The field data can be compared with pre-calculated theoretical curves for given earth models (curve matching process).

2. The wide spectrum of resistivity values measured under different field conditions and the large electrode spacings necessary for exploring the ground to moderate depths makes the use of logarithmic coordinates a logical choice.

- When used for interpretation by curve matching, the scale must be identical with that for the master curve set.

- For sounding, the recommended arrangement is Schlumberger for these advantages:

  1. Schlumberger is less sensitive to lateral variations in resistivity since the effect of near surface inhomogeneities in their vicinity (due to soil condition, weathering is constant for all observations).

  2. Schlumberger is slightly faster in field operation and requires less physical movement of electrodes than the normal Wenner array since only the current electrodes must be moved between readings.

  3. In a Schlumberger sounding, the potential electrodes are moved only occasionally, whereas in a Wenner sounding the potential and the current electrodes are moved after each measurement.
4. Schlumberger sounding curves give a slightly greater probing depth and resolving power than Wenner sounding curves for equal (AB) electrode spacing.

5. The manpower and the time required for making Schlumberger soundings are less than that required for making Wenner soundings.

6. Stray currents in industrial areas and telluric currents that are measured with long spreads affect measurements made with the Wenner array more than those made with the Schlumberger array.

7. The effects of near surface, lateral inhomogeneities are less on Schlumberger measurements than on Wenner measurements.

8. Unstable potential difference is created upon driving two metal stakes into the ground. This difference becomes constant after about 5-10 minutes. Fewer difficulties of this sort are encountered with the Schlumberger array than with Wenner array.

The advantages of Wenner over Schlumberger

1. Wider spacing of the potential electrodes with Wenner results in larger potential differences. This translates into less severe instrumentation requirements for a given depth capability.

2. The relative simplicity of the apparent resistivity formula \( \rho_a = 2\pi a (\Delta V/I) \).

3. The relatively small current values are necessary to produce measurable potential difference.
4. The availability of a larger album of the theoretical master curves for two, three and four layer earth models.

- The above comparison indicates that it is advantageous to use the Schlumberger array rather than the array for making electrical resistivity soundings.

b) **Electrical horizontal profiling (mapping or trenching).**

- In horizontal profiling, a fixed electrode spacing is chosen (depends on the results of the electrical sounding) and the whole electrode array is moved along a profile after each measurement is made to determine the horizontal variation of resistivity.

- It is useful in mineral exploration where the detection of isolated bodies of anomalous resistivity is required.

- The value of apparent resistivity is plotted at the geometric center of the electrode array.

- Maximum apparent resistivity anomalies are obtained by orienting the profiles at right angles to the strike of the geologic structures.
Representation of profiling data

Fig 22: Horizontal profiles over a buried stream channel using two electrode spacings

- The above figure shows an example of data presentation for resistivity profiling using different electrode spacing. It is recommended that at least two different electrode spacing be used in order to distinguish the effects of shallow geologic structures from the effects of deeper ones. The data points have been connected by a smooth curve. Some interpreters may prefer to connect the data points by straight lines.

- We note that the horizontal scale must always be linear. The vertical scale is shown as logarithmic, but a linear scale may also be used.

- The following figure shows a resistivity profiling data presented as a contour plot. The circles represent locations at which the readings were taken.
The results are presented as apparent resistivity profiles or apparent resistivity maps or both.

Sub-surface imaging or two dimensional electrical tomography is used for very high resolution in the near-surface in archaeological, engineering and environmental investigation.

Fig 24: Horizontal profile and interpretation over a shallow gravel deposits
Fig 25: Apparent resistivity map using Wenner array
- Schlumberger electric profiling

The two current electrodes (AB) remains fixed at a relatively large distance (1-6 Km) and the potential electrodes (MN) with a small constant separation are moved along the middle third of the line AB.

(\(\rho_a\)) is calculated for each position that the mobile pair of potential electrodes takes.

At the end of the profile line the Schlumberger setup is transferred on the adjacent line and so on until the area to be investigated has been covered.

This arrangement is sometimes called the Schlumberger AB profiling or Brant array.

- Rectangle resistivity profiling

Fig 27: Rectangle of resistivity
- It is a modification of the Brant array in which the potential electrodes are moved not only along the middle third of the line AB but also along lines laterally displaced from and parallel to AB.

- The lateral displacement of the profile from the line AB may be as much as AB/4.

- The interval MN is kept comparatively small (AB/50 to AB/25) so that make a larger number of measurements within a given rectangle without moving the current electrodes.

- **Wenner electric profiling**

  - The four electrodes configuration with a definite array spacing (a) is moved as a whole in suitable steps, say 10 – 20m along a line of measurement.

  - The interpretation of horizontal profiling data is generally qualitative and the primary value of the data is to locate geologic structures such as buried stream channels, veins and dikes.

- **Constant – separation traversing (CST)**

  ![Diagram](image.png)

  **Fig 28:** A constant separation traverse using a Wenner array with 10m electrode spacing over a clay filled in limestone
- CST uses Wenner configuration with a constant electrode separation and discrete station interval along the profile.

- The entire array is moved along a profile and values of apparent resistivity determined at discrete intervals along the profile.

- Example: suppose electrode separation is 10 meters, we can make a resistivity measurement at station interval of 5 meter or even 2 meters along the array using additional electrodes.

- Instead of uprooting the entire sets of electrodes, the connections are moved quickly and efficiently to the next electrode along the line, i.e. 5m down along the traverse.

![Diagram](image)

**Fig 29**

- This provides a CST profile with electrode separation of 10m and station interval of 5m.

- The values of apparent resistivity are plotted on a linear graph as a function of distances along the profiles.

- Variations in the magnitude of apparent resistivity highlight anomalous areas along the traverse.
Dipole – Dipole mapping (profiling)

Fig 30: example of the measurement sequence for building up a resistivity pseudo-section

- A collinear dipole – dipole configuration can be moved as a whole along lines parallel to the array keeping the values of (a) and (n) fixed.

- Measurements are made along a profile with a selected (a) (60m) and with n=1.

- A discrete set of four electrodes with the shortest electrode spacing (n=1) is addressed and a value of apparent resistivity obtained.

- Successive sets of four electrodes are addressed, shifting each time by one electrode separation laterally.

- Once the entire array has been scanned, the electrode separation is doubled (n=2) and the process repeated until the appropriate number of levels has been scanned.
- With multi-core cables and many electrodes, the entire array can be established by one person.

- The horizontal resolution is defined by the inter-electrode spacing and the vertical resolution by half the spacing. For example, using a 2m inter-electrode spacing, the horizontal and vertical resolutions are 2m and 1m, respectively. For the pseudo-section display.

- In plotting the measurements on paper, lines making an angle of 45° with the line representing the profile are drawn from the centers of the current and potential dipoles in opposite direction and the value of apparent resistivity obtained for that position of the array is plotted at the intersection of these two lines as shown in the above figure.

- The values of apparent resistivity obtained from each measurement are plotted on a pseudo-section and contoured.

- The measurements along the same profiles are repeated for n=2,3,….. and plotted in a similar way.

- It is easy to see that the measurements for n =2 in such a plot will appear along a line below the line on which those for n=1 appear, those for n=3 will be plotted along a line still deeper and so on.

- Contours of equal apparent resistivitites are then drawn on this plot.

- The picture thus obtained is called a vertical pseudo-section of the ground because measurements for a larger value of (n) may be supposed to contain more information about deeper in-homogeneities than those for a small (n).
Special requirements for Schlumberger measurements

- The sounding must be started with small (MN) compared to (AB). MN must never exceed 1/5 AB (MN < 1/5 AB). The field procedure consists in expanding AB while holding MN fixed.

- This process yields a rapidly decreasing potential difference across MN, while exceeds the measuring capabilities of the instrument.

- At this point, a new value for MN must be established, typically 2-4 times larger than the preceding value and the survey is continued.

- The last one or two AB values should be duplicated with the new MN values. The same process may need to be repeated later. To illustrate this read the following example:

Suppose that the survey started with MN = 0.3 meter and AB= 1, 1.47, 2.15, 3.16, 4.64, 6.81, 10, 14.7 (each next value is obtained by matching the preceding one by 1.47 (10^{1/6}). At AB=14.7, we suppose that the instrument sensitivity has declined and a large value of (MN) is required. We increase it to MN = 1.0 meter. Repeat the last two AB values and continue: AB = 10, 14.7, 21.5, 31.6, 46.4, 68.1. At AB = 68.1 we suppose that we must again change (MN) to 3.0 meter. The process continues in this way until the survey is completed.

- The change in MN values during the progress of the sounding introduces a problem for interpretation (un-smoothed curve). The problem arises because the apparent resistivity values turns out to differ slightly for the same AB-value when MN is changed as shown in this figure:
For a down going segment of the resistivity sounding curve, the new value for apparent resistivity will be larger than the old value.

For an up going segment, the new value will be smaller than the old values.

For interpretation, the segmented curve must be converted to a single smooth curve. This process is shown by the dotted lines.

The smooth curve follows the right hand portion of each segment. The smoothed line (dotted line) is drawn below the data points for a down going segment and above the data points for an up going segment. The final dotted curve provides the field curve for interpretation.
Fig. 33: Correct displacement on a Schlumberger sounding curve and method of smoothing

- In the Wenner procedure of electric profiling the four configuration with a definite array spacing, \(a\), is moved as a whole in suitable steps, say 10 – 20m along a line of measurement.

- The choice of array spacing depends on the depth of the anomalous resistivity features to be mapped.
RECOMMENDATIONS FOR FIELD MEASUREMENTS

1. The field measurements must be carefully taken with reliable instruments. These instruments must measure potential and current (or their ration) to high accuracy (order of 1%). A difficulty to be avoided is current leaksge into the ground from poorly insulated current cables.

- Perhaps the largest source of field problems is the electrode contact resistance. Resistivity method rely on being able to apply current into the ground. If the resistance of the current electrodes becomes anomalously high, the applied current may fall to zero and the measurement will fail. High contact resistances are particularly common when the surface material into which the electrodes are implanted consists of dry sand, boulders, gravel, frozen ground, ice. There are two methods to overcome the high resistance of the electrodes and reduce electrode resistance:

a. Drive the electrodes down to moist earth if possible. In some areas this may be a few centimeters and in other areas a meter or more. Wet the current electrodes with water or saline solution, sometimes mixed with bentonite.

b. Use multi-electrodes. Two or three extra electrodes can be connected to one end of the current-carrying cable so that the electrodes act as resistances in parallel. The total resistance of the multi electrode is thus less than the resistance of any one electrode. However, if this method is used, the extra electrodes must be implanted at right angles to the line of the array rather than along the direction of the profile. If the extra electrodes are in the line of the array, the geometric factor may be altered as the inter-electrode separation \((C_1 - P_1 - P_2 - C_2)\) is effectively changed. This problem is only acute when
the current electrode separation is small. Once the current electrodes are sufficiently far apart, minor anomalies in positioning are insignificant.

c. Ideally, a VES array should be expanded along a straight line. If it curves significantly and/or erratically and no correction is made, cusps may occur in the data owing to inaccurate geometric factors being used to calculate apparent resistivity values.

Fig 34: Any number of additional electrodes acts as parallel resistances and reduces the electrode contact resistance.

2. Electrode resistance should be kept low because:

   a. Larger values of electrode resistances will decrease the instrument sensitivity and may introduce spurious potentials.

   b. High resistance at the current electrodes will appear as low total current flow.
c. High resistance at the potential electrodes will appear as low sensitivity and ambiguity in taking the potential reading.

3- For profiling, the recommended value of (a) equals the depth of interest multiplied by a factor of approximately 1.5 – 2. The profile should be repeated with different values of (a).

4- For sounding, successive electrode spacing must be equally spaced on a logarithmic scale of distance. This is because a widely used method for interpretation requires presentation on logarithmic graph paper.

5- The number of data points per decade (one decade equals a factor of 10) should be at least six points. To achieve this value, each value of electrode spacing must equal the previous value multiplied by \(10^{1/6} = 1.47\). For example: if the smallest electrode spacing equals 1 meter, then successive values would be 1.47, 2.15, 3.16, 4.64, 6.81, 10.00, 14.68, etc.....

6- For sounding to a desired depth of investigation (D), the recommended range of electrode spacing extends from a minimum of D/5 to a maximum of 4-6 times D.

7- A VES array should be expanded along a straight line. If it curves significantly and no correction is made, cusps may occur in the data owing to inaccurate geometric factors being used to calculate apparent resistivity values.

8- For quantitative interpretation, the data should span at least 2 decades and preferably 2.5 to 3 decades.
GEOELECTRIC SECTIONS AND GEOELECTRIC PARAMETERS

- The geoelectric section describes the electrical properties of a sequence of layered rocks.

- A geologic section differs from a geoelectric in that the boundaries between geologic layers do not necessarily coincide with the boundaries between layers characterized by different resistivities.

- In the geoelectric sections, the boundaries between layers are determined by resistivity contrasts rather than by the combination of factors used by the geologists in establishing the boundaries between beds (such as fossils, textures,....).

- Example 1, when the salinity of ground water in a given type of rock varies with depth, several geologic layers may be distinguished within a lithologically homogeneous rock.

- Example 2, in an unconfined sandstone aquifer, there is a capillary zone above the water table making the boundary from "dry" to "saturated" a rather diffuse one.

- In the opposite situation layers of different lithologies or ages or both, may have the same resistivity and thus form a single geologic layer.

- It is also common that rocks covering a long period geologically may be uniform electrically, and all can be combined into a single unit in the geoelectric section.

- A geoelectric layer is described by two fundamental parameters: its resistivity (\( \rho \)) and its thickness (\( h \)).
Other geoelectric parameters are derived from its resistivity and thickness. These are (Dar Zarrouk parameters, which were called by Maillet 1947 after a place near Tunis where he was a prisoner of war):

1) Longitudinal unit conductance \( S = h / \rho = h / \sigma \)
2) Transverse unit resistance \( T = h \rho \)
3) Longitudinal resistivity \( \rho_L = h / S \)
4) Transverse resistivity \( \rho_t = T / h \)
5) Anisotropy \( \lambda = \sqrt{\rho_t / \rho_L} \)

For an isotropic layer \( \rho_t = \rho_L \) and \( \lambda = 1 \).

These secondary geoelectric parameters are particularly important when they are used to describe a geoelectric section consisting of several layers.

For "n" layers, the total longitudinal unit conductance is:
\[
S = \sum_{i=1}^{n} \frac{h_i}{\rho_i} = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} + \ldots + \frac{h_n}{\rho_n}
\]

The total transverse unit resistance is:
\[
T = \sum_{i=1}^{n} h_i \rho_i = h_1 \rho_1 + h_2 \rho_2 + \ldots + h_n \rho_n
\]

The average longitudinal resistivity is:
\[
\rho_L = H / S = \frac{\sum_{i=1}^{n} h_i}{\sum_{i=1}^{n} \frac{h_i}{\rho_i}}
\]

The average transverse resistivity is:
\[
\rho_t = T / H = \frac{\sum_{i=1}^{n} h_i \rho_i}{\sum_{i=1}^{n} h_i}
\]
The anisotropy is:

\[ \lambda = \sqrt{\frac{\rho_t}{\rho_L}} = \sqrt{\frac{S}{T/H}} \]

The parameters \( S, T, \lambda, \rho_t \) and \( \rho_L \) are derived from consideration of a column of unit square cross-sectional area (1 X 1 meter) cut out of a group of layers of infinite lateral extension as follows:

\[ T = \rho_1 h_1 + \rho_2 h_2 + \ldots \]

Fig 35: Columnar prism used in defining geoelectric parameters of a section

If current flows vertically only through the column, then the layers in the column will behave as resistors connected in series, and the total resistance of the column of unit cross-sectional area will be:

\[ R = R_1 + R_2 + R_3 + \ldots + R_n \]

Or

\[ R = \rho_1 \left( \frac{h_1}{1 \times 1} \right) + \rho_2 \left( \frac{h_2}{1 \times 1} \right) + \ldots + \rho_n \left( \frac{h_n}{1 \times 1} \right) \]

\[ R = \sum_{i=1}^{n} \rho_i h_i = T \]

The symbol "T" is used instead of "R" to indicate that the resistance is measured in a direction transverse to the bedding.
If the current flows parallel to the bedding, the layers in the column will behave as resistors connected in parallel and the conductance will be:

\[ S = \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \]

Or

\[ S = \frac{1 \times h_1}{(\rho_1 \times 1)} + \frac{1 \times h_2}{(\rho_2 \times 1)} + \ldots (1 \times h_n)/(\rho_n \times 1) \]

\[ S = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} + \ldots \frac{h_n}{\rho_n} \]

The dimensions of the longitudinal unit conductance are \( m/\text{ohm-m} = 1/\text{ohm} = \text{mho} \).

Example:

Assume that a geoelectric unit consists of an alternating series of beds with a total thickness of 100m. The individual beds being isotropic, one meter thick and resistivities alternating between 50 and 200 ohmm.

\[ T = \Sigma \rho_i h_i = 50 \times 50 + 200 \times 50 = 12,500 \text{ ohm-m}^2 \]
\[ \rho_t = \frac{T}{H} = \frac{12,500}{100} = 125 \text{ ohm-m} \]
\[ S = \Sigma \sigma_i h_i = 50 \times 1/200 = 1.25 \text{ mhos} \]
\[ \rho_L = \frac{H}{S} = \frac{100}{1.25} = 80 \text{ ohm-m} \]
\[ \lambda = \sqrt{\frac{\rho_t}{\rho_L}} = \frac{125}{80} = 1.25 \]

Many igneous and metamorphic rocks may show a layeres or zoned electrical structure similar to the electrical layering found in sedimentary rocks.

Volcanic rocks frequently are layered.
TYPES OF ELECTRICAL SOUNDING CURVES OVER HORIZONTALLY STRATIFIED MEDIA

- The form of the curves obtained by sounding over a horizontally stratified medium is a function of the resistivities and thicknesses of the layers as well as of the electrode configuration.

- **Homogeneous and isotropic medium (One layer medium):** if the ground is composed of a single homogeneous and isotropic layer of infinite thickness and finite resistivity, the apparent resistivity curve will be a straight horizontal line whose ordinate is equal to the true resistivity ($\rho_t$) of the semi-infinite medium.

Fig 36: Single layer medium

**Two-layer medium**

Fig 37: Two-layer Schlumberger curves
If the ground is composed of two layers, a homogeneous and isotropic first layer of thickness \( h_1 \) and resistivity \( \rho_1 \) underlain by an infinitely thick substratum \( h_2 = \infty \) of resistivity \( \rho_2 \) then the sounding curve begins at small electrode spacing with a horizontal segment \( \rho' = \rho_1 \).

As the electrode spacing is increased, the curve rises or falls depending on whether \( \rho_2 > \rho_1 \) or \( \rho_2 < \rho_1 \) and on the electrode configuration used.

At electrode spacings much larger than the thickness of the first layer, the sounding curve asymptotically approaches a horizontal line whose ordinate is equal to \( \rho_2 \).

The electrode spacing at which the apparent resistivity \( \rho' \) asymptotically approaches the value \( \rho_2 \) depends on three factors:

1) The thickness of the first layer \( h_1 \)
2) The value of the ratio \( \rho_2 / \rho_1 \)
3) The type of electrode array used in making the sounding measurements.

The dependence of the electrode spacing on the thickness of the first layer is fairly obvious. The larger the thickness of the first layer, the larger the spacing required for the apparent resistivity to be approximately equal to the resistivity of the second layer.

For most electrode array, including the Schlumberger, Wenner, dipole–dipole, when \( \rho_2 / \rho_1 > 1 \), larger electrode spacings are required for \( \rho' \) to be approximately equal to \( \rho_2 \) than when \( \rho_2 / \rho_1 < 1 \) (see the above figure).
Three-layer medium

Fig 38: Example of the four types of three-layer Schlumberger sounding curves

- If the ground is composed of three layers of resistivities $\rho_1, \rho_2, \rho_3$ and thicknesses $h_1, h_2, \text{and } h_3 = \infty$, the geoelectric section is described according to the relation between the values of $\rho_1, \rho_2, \rho_3$.

- There are four possible combinations between the values of $\rho_1, \rho_2, \rho_3$. These are:
  
  1) $\rho_1 > \rho_2 < \rho_3$ ....... H-type curve (minimum type)
  2) $\rho_1 < \rho_2 < \rho_3$ ....... A-type curve (ascending type)
  3) $\rho_1 < \rho_2 > \rho_3$ ....... K-type curve (maximum type)
  4) $\rho_1 > \rho_2 > \rho_3$ ....... Q-type curve (descending type)
Fig 39: Types of the Sounding curves

- Types H and K have a definite minimum and maximum, indicating a bed or beds of anomalously low or high resistivity respectively at intermediate depth.
• Types A and Q shows fairly uniform change in resistivity, the first increasing the second decreasing with depth.

**Multilayer – medium**

![Diagram showing various types of Schlumberger sounding curves]

Fig 40: Examples of the eight possible types of Schlumberger sounding curves for four-layer Earth models

• If the ground is composed of more than three horizontal layers of resistivities \( \rho_1, \rho_2, \rho_3, \ldots, \rho_n \) and thicknesses \( h_1, h_2, h_3, \ldots, h_n = \infty \), the geoelectric section is described in terms of relationship between the resistivities of the layers and the letters H, A, K and Q are used in combination to indicate the variation of resistivity with depth.

• In four – layer geoelectric sections, the types of the three-layer curves may be combined to give the following eight possible relations between \( \rho_1, \rho_2, \rho_3, \) and \( \rho_4 \):
1. \( \rho_1 > \rho_2 < \rho_3 < \rho_4 \) ………… HA – type curve
2. \( \rho_1 > \rho_2 < \rho_3 > \rho_4 \) ………… HK – type curve
3. \( \rho_1 < \rho_2 < \rho_3 < \rho_4 \) ………… AA – type curve
4. \( \rho_1 < \rho_2 < \rho_3 > \rho_4 \) ………… AK – type curve
5. \( \rho_1 < \rho_2 > \rho_3 < \rho_4 \) ………… KH – type curve
6. \( \rho_1 < \rho_2 > \rho_3 > \rho_4 \) ………… KQ – type curve
7. \( \rho_1 > \rho_2 > \rho_3 < \rho_4 \) ………… QH – type curve
8. \( \rho_1 > \rho_2 > \rho_3 > \rho_4 \) ………… QQ – type curve

- Examples of Schlumberger electrical sounding curves for these eight types of four-layer models are shown in the above figure

- For a five-layer geolectric section there are 16 possible relationships between the resistivities, therefore there are 16 types of five-layer electrical sounding curves.

- Each of these 16 curves may be described by a combination of three letters (i.e. HKH curve in which \( \rho_1 > \rho_2 < \rho_3 > \rho_4 < \rho_5 \)).

- In general: an n-layer section is described by (n-2) letters.
INTERPRETATION OF RESISTIVITY SOUNDING DATA

- Vertical sounding curves can be interpreted qualitatively using simple curve shapes, semi-quantitatively with graphical model curves or quantitatively with computer modeling.

- The last method is the most rigorous but there is a danger with computer methods to over-interpret the data.

- Often a noisy field curve is smoothed to produce a graph which can then be modeled more easily.

- In this case, the interpreter spends much time trying to obtain a perfect fit between computer – generated and field curves.

- Near-surface layers tend to be modeled more accurately than those at depth because field data from shorter electrode separations tend to be more reliable than those for very large separation, owing to higher signal – to – noise ratios.

Plotting

- The first step in the interpretation of VES measurements is to plot these in a graph.

- If we have a Wenner sounding the measured ($\rho_a$) is plotted on the y-axis and the electrode separation (AB/2) on the x-axis.

- The data of a Schlumberger sounding are plotted with $L = AB/2$ along the x-axis. This data should be drawn on a double logarithmic paper.
In the interpretation of resistivity sounding we note that the earth is assumed to consist of uniform layers, separated horizontal interfaces. The parameters of this model are:

1) Number of layers
2) Resistivity values for each layer, and
3) Thickness for each layer

The interpretation process consists of determining numerical values for these parameters.

The interpretation will be guided by the information from geologic studies, drill holes, road cuts, etc..

Qualitative Interpretation

The qualitative interpretation of sounding data involves the following:

1) Study of the types of the sounding curves obtained and notation of the areal distribution of these types on a map of the survey area.
2) Preparation of apparent resistivity maps. Each map is prepared by plotting the apparent resistivity value, as registered on the sounding curve, at a given electrode spacing (common to all soundings) and contouring the results.

![Apparent Resistivity Map](image)

Fig 43: Section of apparent resistivity

3) Preparation of apparent resistivity sections. These sections are constructed by plotting the apparent resistivities, as observed, along vertical lines located beneath the sounding stations on the chosen profile. The apparent resistivity values are then contoured. Generally a linear vertical scale is used to suppress the effect of near-surface layers.

4) Preparation of profiles of apparent resistivity values for a given electrode spacing (profiles of the ordinate or abscissa of the values of the minimum points for H-type curves, profiles of the ordinate or abscissa of the maximum point for K-type, profiles of $\rho_L$ values and profiles of "$S" and "T" values.

- These maps, sections and profiles constitute the basis of the qualitative interpretation which should proceed quantitative interpretation of the electrical sounding data.
It should be noted that an apparent resistivity map for a given electrode spacing does not represent the areal variation of resistivity at a depth equal to that electrode spacing. It merely indicates the general lateral variation in electrical properties in the area.

For example, an area on the map having high apparent resistivity values may correspond to a shallow high resistivity bedrock, it may indicate thickening in a clean sand and gravel aquifer saturated with fresh water, or it may indicate the presence of high resistivity gypsum or anhydrite layers in the section.

**Determination and use of Total Transverse Resistance, T, from Sounding Curves**

- In three-layer sections of the K type, the value of transverse resistance \( T_2 \) of the second layer can be determine approximately from a Schlumberger sounding curve by multiplying the ordinate value of the maximum point \( \rho_{s_{\text{max}}} \) by the corresponding abscissa value of \( AB/2 \) (Kunetz, 1966).

- The total transverse resistance of the upper two layers \( T \) can be determined by applying the following formula:

\[
T = T_1 + T_2 = \rho_1 h_1 + \rho_2 h_2
\]

Or by graphical technique as follows:
Fig 44: Graphical determination of total transverse resistance from a K-type Schlumberger sounding curve

1. The intercept of a straight line tangent to the Schlumberger sounding curve and inclined to the abscissa axis at an angle of 135° (or -45°) with the horizontal line for \( \rho = 1 \) ohm-m is approximately equal to \( T \) (see the above figure).

2. When the value of "T" increases from one sounding station to the next, this generally means that the thickness of the resistive layer in the section also increases.

3. The increase in "T" might be caused also by an increase in the resistivity values.

- Example: a north-south profile of graphically determined values of total transverse resistance east of Minidoka, Idaho is an excellent qualitative indication that the Snake River basalt increases in thickness appreciably from south to north.
Fig 45: Profile of total transverse resistance values, \( T \), in Ohmm squared. High values indicate thickening of basalt layers

**Determination of Total longitudinal Conductance (S) from Sounding Curves.**

Fig 46: Graphical determination of total longitudinal conductance, \( S \), from an H-type Schlumberger sounding curve
In H, A, KH, HA and similar type sections the terminal branch on the sounding curve often rises at an angle of 45°.

This usually indicates igneous or metamorphic rocks of very high resistivity (> 1000 ohm-m).

However, in the presence of conductive sedimentary rocks saturated with salt water (ρ < 5 ohm-m) the so-called "electric basement" of high resistivity rocks may correspond to sandstone or limestones having resistivities of only 200 – 500 ohm-m.

The total longitudinal conductance "S" is determined from the slope of the terminal branch of a Schlumberger curve, rising at an angle of 45° (here called the S-line).

The value of "S" is numerically equal to the inverse of the slope of this line (Kalenove, 1957; Keller and Frischknecht, 1966)

It is usually determined very quickly, by the intercept of the extension of the S-line with the horizontal line, ρ =1 ohm-m.

Increases in the value of "S" from one sounding station to the next indicate an increases in the total thickness of the sedimentary section, a decrease in average longitudinal resistivity (ρL) or both.

Distortion of sounding curves by extraneous influences

Electrical sounding curves may be distorted by:
1. Lateral in-homogeneities in the ground.
2. Errors in measurements.
3. Equipment failure.
A - Formation of Cusps:

- The formation of a cusp on a Schlumberger sounding curve is caused by:

  1. A lateral heterogeneity which may be a resistive lateral in-homogeneity in the form of a sand lens and a conductive in-homogeneity in the form of a buried pipe or a clay pocket.

  2. A current leakage from poorly insulated cables.

  3. Electrode spacing errors.

  4. Errors in calculation.

- If a conductive clay lens is present, for example, then when a current is applied from some distance away from it, the lines of equipotential are distorted around the lens and the current flow lines are focused towards the lens. The potential between P and Q is obviously smaller than that measured between R and S which are outside the field of effect of the lens (see the above figure).

- The apparent resistivity derived using this value of potential is lower than that obtained had the lens not been there, hence the occurrence of a cusp minimum.

- If the lens has a higher resistivity than the host medium, the current flow lines diverge and the potential between P and Q becomes anomalously high and results in a positive cusp.
Fig. 47: Distortion of sounding curves by cusps caused by lateral in-homogeneities.

**B- Sharp maximum**

Fig 48: Example of a narrow peak on a K-type curve, caused by the limited lateral extent of a resistive middle layer.
- The maximum or peak value on a K-type sounding curve is always gentle and broad and should never have a sharp curvature.

- The formation of a sharp peak is indicative of the limited lateral extent of the buried (middle) resistive layer.

C- Curve discontinuities

Two types of discontinuities are observed on Schlumberger sounding curves:

1- The first is observed when the MN spacing is enlarged with AB constant. This indicates a lateral in-homogeneity of large dimensions (see the above figure).

This type of discontinuity may indicate:
- current leakage.
- electrode spacing errors
- the input impedance of the potential difference measuring device is not sufficiently high.

When the discontinuity are not sever, the curve can be corrected easily by shifting the distorted segment of the curve vertically to where it should be.
2- The second type is less common and occurs during the expansion of the (AB) spacing. The curve is displaced downward.

- This type of discontinuity is caused by a narrow, shallow dike like structure which is more resistance than the surrounding media and whose width is small in comparison to the electrode spacing.

- The abscissa (x-axis) value at which the discontinuity occurs is equal to the distance from the sounding center to the dike like structure.

Fig 50: Examples of discontinuities on Schlumberger curves caused by a near vertical dike like structure
QUANTITATIVE INTERPRETATION

- Several methods are used in the quantitative interpretation of electrical sounding curves. These methods are classified as analytical methods, semi-empirical methods, and empirical methods.

Analytical Methods (Logarithmic curve matching)

- The analytical methods are based on the calculation of theoretical sounding curves that match the observed curves.

- A much more accurate and dependable method of interpretation in electric sounding involves the comparison of field curves with a set of theoretically calculated master curves assuming that the model relates to a horizontally stratified earth and that deeper layers are thicker than those overlying.

- Master curves are calculated assuming that $\rho_1 = 1$ and $h_1=1$ m and plotted on double logarithmic papers.

- There are several catalogues of theoretical master curves calculated for a variety of Earth structures, most of which represent horizontally stratified media.

- It is only practical to use the master curves method for up to four layers. If more layers are present, the graphical approach is inaccurate.

- Three and four–layer models can also be interpreted using master curves for two layers with the additional use of auxiliary curves.
- The use of a high speed digital computer is almost always necessary for the calculation of theoretical sounding curves.

- Before interpretation is made with the master sets for horizontal layers, the interpreter must be satisfied with the form of the sounding curve, in that it is sufficiently smooth and not severely distorted by sharp cusps or discontinuities.

- A certain amount of smoothing generally is required. The type of curve (such as H, A, Q, HA, HK) and the minimum number of layers it seems to represent can be determined by visual inspection.

Fig 51: Smoothing of VES curves
Two-layer Interpretation

- Two-layered stratification may be of two types:
  1. $\rho_2 > \rho_1$ (unconsolidated overburden laying on a bedrock)
  2. $\rho_2 < \rho_1$ (poor conducting alluvium laying over a better conducting sand or a clay formation).

- Two-layer master curves are used to interpret the resistivity data as follows:

1. The field data ($\rho_a$ and $AB/2$) are plotted on a transparent sheet of double logarithmic paper which has exactly the same scales as the graph paper on which a set of theoretical curves have been plotted.

2. The sheet with the field curve is laid over the sheet with the two layer theoretical curves. Move the transparent paper up, down, right or left (maintaining the coordinate axes of the two sheets parallel) until a best fit of the field curve against one of the theoretical curves is obtained.

![Two-layer master curves for Schlumberger and Wenner arrays](image)

Fig 52: Two-layer master curves for Schlumberger and Wenner arrays
3. Occasionally the field curve may have to be matched by interpolation between two of the master curves.

4. Determine the position of the cross, which is the origin of coordinates of the theoretical curve and trace it on the sheet of the field curve.

5. The abscissa value \((AB/2)\) of the "cross" equals the thickness of the first layer and the ordinate value \((\rho)\) of the "cross" equals the true resistivity, \(\rho_1\), of the first layer.

6. Determine the resistivity of the second layer \((\rho_2)\) by tracing the asymptote to the theoretical two-layer curve.

7. The trace of the asymptote to \(\rho_2\) on the field sheet equals the true resistivity, \(\rho_2\), of the second layer.

8. The value of \((\mu)\) corresponding to the field curve is read from the master curve. If no good match is obtained the value of \((\mu)\) is obtained by interpolation between two adjoin master curves.

9. Using the relation \(\mu = \rho_2/\rho_1\), \(\rho_2\) is calculated.

10- The same procedure could be used either in Wenner or Schlumberger array, but each array has its own master curves.
Fig 53: Two-layer master set of sounding curves for the Sclumberger array

Fig 54: Interpretation of a two-layer Schlumberger curve ($\rho_2/\rho_1=5$)
Interpretation of two-layer curves by asymptotes

Fig 55: Example of \((\rho_2)\) and \((Z)\) from the 45° asymptote

- The master curves are not necessary in this case.

- In case that the lower bed has a very large resistivity (i.e. insulator) we have seen that the characteristic two-layer curve becomes a straight line for large electrode spacing.

- The extreme right – hand portion of the sounding curve will approach asymptotically a line has a slope of 45°, for all of the arrays considered since we have made \((\rho_1\) and \(Z\) or \(h\) unity).

- After plotting the field profile on log – log paper, a straight edge is placed horizontally as a best fit along the left – hand portion of the curve.

- The hypotenuse of a 45° triangle is fitted to the sloping part of the curve on the right – hand side of the profile.
- The interface depth (z or h) can then be found on the horizontal axis from the intersection of the triangle and the horizontal straight edge.

- The ratio of spacing to apparent resistivity for any point along the line rising at an angle of 45° will be exactly "S" (the conductance of all the rocks above the insulating layer:

\[ S = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} + \ldots \]

- The asymptote method may also be used even when the maximum spacing has not been large enough to establish that the bottom layer has a very high resistivity.

Fig 56: Asymptote method for estimating minimum depth

- In this case the 45° triangle is placed to intersect the point of maximum spacing. The depth estimate can only be a minimum.
Three-layer Interpretation

- Determine the type of three-layer curves (H, A, K, Q) by inspection and select the applicable set of theoretical master curves.

- The problem of interpretation of three-layer curves is slightly more complicated because of the increased number of parameters ($\rho_1$, $\rho_2$, $\rho_3$, $h_1$, and $h_2$) as shown in the following figure:

![Three-layer master curves family on bottom sheet](image)

Fig 57: Example of three-layer curve matching

- The interpretation procedure for three-layer curves is more or less similar to that for two-layer curves. In short, the procedure consists of:

  1. Matching the left-hand part of the three-layer curves with a two-layer master curve. This enables the determination of $(\rho_1, \rho_2, h_1)$. 
2. When \((\mu = \rho_2/\rho_1)\) and the type of curve are known, the corresponding master curve sheet can be selected from the album of three-layer master curves, and by matching this with a standard curve the other parameters can be determined.

- Better interpretation generally are obtained by enveloping the field curve between two three-layer curves having the same value of \(\mu_1 = \rho_2/\rho_1\) and the same value of \(\nu = h_2/h_1\), different values of \(\mu_2 = \rho_3/\rho_1\) (see the following figure).

Fig 58: Interpretation of a three-layer Schlumberger H-type curve

- If \(\mu_2 = \rho_3/\rho_1\) for the field curve and the theoretical curve are equal, then **complete curve matching** may be attained.

1. Maintaining parallelism between the axes of the field curve and the theoretical curve, determine the position of the cross on the field curve.
2. Note the value of $\nu_1 = h_2 / h_1$ designating the theoretical curve and note the values $\mu_1 = \rho_2 / \rho_1$ and the value $\mu_2 = \rho_3 / \rho_1$.

3. Knowing $h_1$ and $\rho_1$ from the abscissa and ordinate of the cross, the values of $\rho_2$, $h_2$ and $\rho_3$ can be calculated from the values of $\mu_1 = \rho_2 / \rho_1$, $\nu_1 = h_2 / h_1$ and $\mu_2 = \rho_3 / \rho_1$ respectively.

4. If a satisfactory match between the field curve and a theoretical three-layer curve is impossible, then either the curve represents more than three layers, or it is a three-layer curve with a large value of $\mu_1 = \rho_2 / \rho_1$, $\nu_1 = h_2 / h_1$ and $\mu_2 = \rho_3 / \rho_1$ that are not in the album.

5. The interpretation then is made using the two-layer curves in conjunction with auxiliary point diagrams.

**Four-layer (or more) interpretation (by the auxiliary point method) or partial curve matching**

- In practice, especially with large spacings, four or more layers may be distinctly reflected on the curve.

- The maximum number of layers detected by the curve with the electrode spacing $AB/2$ of as much as 10,000 m generally does not exceed eight layers.

- The auxiliary point method depends on the principle of reduction.

- Consider a prism of unit cross section with a thickness (h) and resistivity ($\rho$).
The transverse resistance (T) normal to the face of the prism and the longitudinal conductance (S) parallel to the face of the prism are given by:

\[ T = \rho h \quad (1) \]
\[ h = \frac{T}{\rho} \]

\[ S = \frac{h}{\rho} \quad (2) \]
\[ \rho = \frac{h}{S} \]

From (1) & (2)
\[ h = \frac{TS}{h} \]
\[ h^2 = TS \]
\[ h = \sqrt{TS} \]
\[ \rho = \sqrt{\frac{T}{S}} \]

From equation (1) we get:
\[ \log \rho = - \log h + \log T \]

This equation defines a straight line inclined at an angle 135° to the h-axis and cutting it at a distance T from the origin (o) it is plotted against (h) on a double logarithmic paper.
• From equation (2)
  \[ \log \rho = \log h + \log S \]

• This equation defines a straight line inclined at an angle (45) to h axis and cut it at the distance (S) as follows:

• The intersection of the two straight lines defines resistivity (\(\rho\)) and thickness (h) of a combination of "S" and "T" as follows:
In case of two layer prism:

\[ T = T_1 + T_2 = \rho_1 h_1 + \rho_2 h_2 \]
\[ S = S_1 + S_2 = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} \]

- If \( \rho_t \) and \( \rho_s \) are transverse and longitudinal resistivities of the block, then:

\[ \rho_t ( h_1 + h_2) = \rho_1 h_1 + \rho_2 h_2 \]
\[ \rho_t = \frac{( \rho_1 h_1 + \rho_2 h_2 )}{( h_1 + h_2)} \]
\[ ( h_1 + h_2)/ \rho_s = \frac{h_1}{\rho_1} + \frac{h_2}{\rho_2} \]
\[ \rho_s = \frac{( h_1 + h_2)}{( \rho_1 + h_2/\rho_2)} \]
\[ \sqrt{\rho_t / \rho_s} = \sqrt{\frac{( \rho_1 h_1 + \rho_2 h_2 ) ( h_1/\rho_1 + h_2/\rho_2)}{( h_1 + h_2)^2}} \]
\[ \sqrt{\frac{\rho_t}{\rho_s}} = \lambda = \text{Coefficient of anisotropy} \]

\[ \lambda = \frac{1}{h_1 + h_2} \sqrt{\left( \frac{\rho_1 h_1 + \rho_2 h_2}{h_1/\rho_1 + h_2/\rho_2} \right)} \]

S, T, \rho_s, \rho_t and \lambda are called geoelectric parameters.

- Suppose that the anisotropic prism is replaced by to homogeneous and isotropic prism of a thickness (h_e) and resistivity (\rho_e) which may be called effective thickness and effective resistivity of the block:

  \[ T = h_e \rho_e = \rho_1 h_1 + \rho_2 h_2 \]
  \[ S = h_e / \rho_e = h_1/\rho_1 + h_2/\rho_2 \]

  \[ h_e = \sqrt{TS} \]

  \[ h_e = \sqrt{\left( \rho_1 h_1 + \rho_2 h_2 \right) \left(h_1/\rho_1 + h_2/\rho_2\right)} \]

  \[ = \lambda \left(h_1 + h_2\right) = \lambda H \]

**The effective thickness** (h_e) = \lambda H

\[ \rho_e = \sqrt{T/S} = \sqrt{\left( \rho_1 h_1 + \rho_2 h_2 \right) / \left(h_1/\rho_1 + h_2/\rho_2\right)} \]

**The effective resistivity** \( \rho_e = \lambda \rho_s \)

- From these two equations we can get h_e, \rho_e. Thus it is possible to transform an isolated two-layer block (each homogeneous and isotropic) into a single homogeneous and isotropic medium.

- In partial curve matching, short segments of a resistivity sounding curves are selected for interpretation using the two – layer master curves.
- As each portion of the curve is interpreted the layers comprising the interpreted portion of the sounding curve are lumped (to put together in a single group) together to form a fictitious uniform layer with a lumped resistivity (equivalent resistivity, $\rho_f$) and an equivalent thickness ($h_f$).

- This fictitious layer is then used in place of the surface layers when the next portion of the curve is analyzed.

- The graphical interpretation of multilayer sounding curves is made by using the two-layer curves and the auxiliary point diagrams as follows:
Fig 59: Hypothetical example of auxiliary point method of VES interpretation

1) A transparent paper on which the sounding curve has been traced is slide on the sheet of the two-layer master curve keeping the respective axes parallel to each other until a reasonably long portion of the first branch of the measured curve coincides with one of the master curves.
2) In the above example, the coinciding master curve is one for which \( \rho_2/\rho_1 = 3 \) (dashed line).

3) The origin \((1,1)\) of the master curves is marked on the tracing paper (circle A with cross). The coordinates of "A" gives \( \rho_1 \) and \( h_1 \).

4) We must now seek a way to combine the first two layers into a single fictitious layer so that the right hand portion of the field data may be interpreted.

5) This can be done by placing the tracing paper on the auxiliary diagram (auxiliary diagram is a system of master curves which has been drawn on a logarithmic scale of the same modulus) with "A" coinciding with the origin of the auxiliary diagram, keeping the respective axes being parallel.

6) The auxiliary curve for which \( \rho_2/\rho_1 = 3 \) is copied on the tracing paper for a sufficient length. This is the curve marked \( \rho_m/\rho_1 = 3 \).

7) The tracing paper is now again sliding over the sheet of the two-layer master curve, keeping the axes parallel and with the origin of the master curves always on the copied curve \( \rho_m/\rho_1 = 3 \), until a reasonably long portion of the descending branch of the field curve (portion 2) coincides with one of the master curves.

8) In the above figure this represents the dashed line \( \rho_3/\rho_m = 0.11 \).

9) The origin of the master curve is again marked on the tracing paper (point B). The coordinates of point "B" give \((h_1 + h_2)\) and \( \rho_m \) which is \( \rho_m \) and then \( h_2 \) and \( \rho_3 \) can be determined knowing \( h_1 \) and from the ratio \( \rho_3/\rho_m = 0.11 \).
10) The tracing is now once more placed on the auxiliary diagram and the curve for $\rho_m/\rho_1 = 0.11$ copied.

11) Again place the tracing paper on the two-layer master curve and slide it keeping the origin point of the master curves always on the copied curve $\rho_m/\rho_1 = 0.11$, until the ascending curve (portion 3) coincides with one of the master curve.

12) This is the dashed curve $\rho_4/\rho_m^2 = 9$ and then the cross of the master curves is copied on the tracing paper (point C). The coordinates of which gives $\rho_m^2$ and $h_1 + h_2 + h_3$.

13) Then $\rho_4$ can be determined from the ratio $\rho_4/\rho_m^2 = 9$ and also $h_3$.

14) The resistivities of the various layers are easily obtained from the ratios: $\rho_2/\rho_1$, $\rho_3/\rho_m^1$, $\rho_4/\rho_m^2$ and the thicknesses from cross (A) gives $h_1$, cross (B) gives $h_1 + h_2$, cross (C) gives $h_1 + h_2 + h_3$ or from the auxiliary curves.

15) There are auxiliary curves for all types of field curves (H, K, A& Q).

16) The thickness of the subsurface layers can be determined graphically using the same family of auxiliary curves. This is done as follows:
Fig 60: Auxiliary curves, type A

a. The dashed curves on the auxiliary curves represent $h_2/h_1$.

b. After the right hand portion of the data has been matched with a theoretical curve, the point B ($\rho_f$, $h_f$) indicating the proper values for the thickness and resistivity of the fictitious layer is marked on the field plot.

c. The auxiliary curves are superimposed a second time on the field plot and the true thickness $h_2$ is found by noting the parameter for the dashed curve which passes through the point (B).
Examples of interpretation for different VES curve types

1- Type H-curve

a- Type H-curve:

Fig 61: Interpretation of three-layer H-type curve by Two-layer master curves and auxiliary curves

a) The field curve is superimposed over the theoretical curves for a single overburden (two-layer curves) and moved around until the first portion of the field data matched with one of the curves.

b) The location of the origin on the theoretical curves is shown as the point (P). The locus of this point on the plot of the field data indicates the resistivity and thickness of the first layer ($\rho_1$, $h_1$).
c) The resistivity of the second layer ($\rho_2$) is deduced from the ratio $\rho_2 / \rho_1$ (the resistivity contrast parameter for the particular theoretical curve which matches with the field data).

d) The point ($P_1$) on the field curve is placed at the origin of the auxiliary curve (H-type) keeping the parallism between the axes.

e) The auxiliary curve corresponding to the resistivity ratio found from the initial match is traced onto the field data and the interpretation is completed as above.

2- Type A-curve

![Auxiliary curves, type A](image)

Fig 62: Auxiliary curves, type A

a) The fictitious layer is thicker than the combination of the top two layers and the factor by which it is thicker is the coefficient of anisotropy for these two layers.

$$h_f = \lambda (h_1 + h_2)$$

b) In use, these curves are super-imposed on a set of the field data in such a way that the horizontal axis at the top of the graph lies along the value found for ($\rho_2$) by the initial curve match and the left vertical axis of the graph passes
through the point \((P_1)\), the origin for the signal overburden curves when the first portion of the field data is matched.

c) The appropriate auxiliary curve (determined by the ratio in resistivity between the first two layers) is then traced onto the field plot.

d) The interpretation is completed as above.

e) After the right hand portion of the data has been matched with a theoretical curve, the point \((P_f)\) indicating the proper value for the thickness and resistivity of the fictitious layer \((h_f, \rho_f)\) is marked on the field plot.

f) The auxiliary curves are super-imposed a second time on the field plot, and the true thickness \((h_2)\) is found by noting the parameter for the dashed curve which passes through the point \((P_f)\).

g) As an example, we shall consider a computed type \((A)\) curve so that we know the correct interpretation. Such a curve is shown in the following figure for a case in which the ratio of resistivities between the three layers is 1:3:10 and the ratio in thickness between the first two layers is 1:5.

Fig 63: Interpretation of type –A curve (left), interpretation of the right hand portion of the curve (right)
h) The location of the origin (P₁) is located for \( \rho_2 / \rho_1 = 3 \).

i) Using the auxiliary A-type curves, the curve with \( \rho_2 / \rho_1 = 3 \) is traced on the field data sheet starting at \( \rho_1 \) and rising asymptotically to a value of 3 \( \rho_1 \).

j) Next the two-layer master curves are placed over the field data and moved around until a match with the right – hand portion of the data is obtained (the origin of the theoretical curves is restricted to lie along the auxiliary curve).

k) The matching curve indicates a resistivity ratio \( \rho_3 / \rho_f = 3.7 \).

l) The locus of the point (Pₗ) gives the thickness and resistivity of the fictitious layer:

\[
\begin{align*}
\rho_f &= 2.7 \, \Omega m \\

h_f &= 6.5 \, m
\end{align*}
\]

m) The correct thickness for the second layer is found by placing the auxiliary A-type curves over the field data a second time and noting which of the dashed curves passes through the point (Pₗ).

n) In this case, the dashed curve has the parameter (5), so the thickness of the second layer is five times the thickness of the first layer (\( h_2/h_1=5 \)).

**Type K-curves**

- In the following figure, the left hand portion of the field data can be fitted with a theoretical curve for a single overburden (i.e. two-layer curve).
The locus of the point (P₁) provides the following:

\[ h_1 = 42\, \text{m} \]
\[ \rho_1 = 235\, \text{Ohmm} \]
\[ \rho_2 = 50\, \rho_1 = 11.700\, \text{Ohmm} \]

Using the following auxiliary curves, type K, the auxiliary curve for a ratio of 50:1 (\( \rho_2 / \rho_1 = 50 \)) is plotted over the field data.
Next the two-layer master curves were superimposed on the data to match the right-hand portion of the data.

The locus \((P_f)\) provides the information:

\[
\begin{align*}
h_f &= 525 \text{ m} \\
\rho_f &= 2500 \text{ Ohmm} \\
\rho_3 &= 1/50 \rho_f = 50 \text{ Ohmm}
\end{align*}
\]

The thickness of the second layer may be determined graphically by placing the auxiliary curves over the field data and noting which of the \(h\)-parametric curves passes through the point \((P_f)\). The second layer has a thickness 2.6 \(h_1\).

\[
\frac{h_2}{h_1} = 2.6 \\
h_2 = 2.6 \times 42 = 109 \text{ m}
\]

**Q-type curve**

As an example, we shall consider Q-type curve in which the second layer has a resistivity \(1/3 \rho_1\) and \(h_2 = 5 h_1\) and \(\rho_3 = 1/10 \rho_1\).

\[
\begin{align*}
\rho_2 &= 1/3 \rho_1 \\
\rho_3 &= 1/10 \rho_1 \\
h_2 &= 5 h_1
\end{align*}
\]

The match between the first portion of this curve and the two-layer master curves gives the location of point \((P_1)\) as follows:

\[
\begin{align*}
h_1 &= 1 \text{ m} \quad \rho_1 &= 1 \text{ ohmm} \\
\rho_2 &= 1/3 \rho_1 = 1/3 \text{ ohmm}
\end{align*}
\]

The appropriate auxiliary curve for \(\rho_2 / \rho_1 = 1:1/3\), is then traced onto the data plot.
Fig 66: Auxiliary curves, type Q

- The two-layer master curves are then matched with the right-hand portion of the data.

- The locus of the point \((P_f)\) provides the information:
  \[
  h_f = 3.4 \text{ m} \\
  \rho_f = 0.33 \text{ Ohmm} \\
  \rho_3 = 0.32 \rho_f = 0.105 \text{ Ohmm}
  \]

- The final step in interpretation is to replace the auxiliary curves over the data plot and note which of the \(h\)-parameter curves passes through the point \((P_f)\) and find the thickness of the second layer \(h_2\).

- The partial curve matching technique works best for type (H) curves and least well for type (A) curves.

- Better results are usually obtained in interpreting type (K) curve than in interpreting type (Q) curves.
DC Resistivity. Step 2 of investigation: The model of inferred resistivities, and its implications for hydrogeology.

<table>
<thead>
<tr>
<th>Layer</th>
<th>Resistivity</th>
<th>Thickness</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>325</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1750</td>
<td>Infinite</td>
</tr>
</tbody>
</table>

- = Observations
- = Model Response
Computer can be used to calculate master curves for vertical electrical soundings obtained using either a Wenner or Schlumberger.

The program synthesizes an apparent resistivity profiles for an n-layered model in which the variables are layer thickness and resistivity.

Model profile can then be compared with the field curves and adjustments to the layering and resistivity values can be made by trial and error to obtain as near correspondence as possible to the field curve.

Some computer packages display both the field and model curves simultaneously and may produce statistical parameters to produce the closeness of the fit.

Optimization of the interpretation can be achieved automatically by successive iterations to reduce the degree of misfit until it falls within a specified and acceptable statistical limit.

The final step of a resistivity interpretation should be to relate each accepted VES model to the unknown local geological and tables of resistivities and rock types can be produced.

**Zohdy Program (ATO)**


The starting model is used to generate a theoretical synthetic sounding curve which is compared with the field data.
Fig 67:

- An iterative process is then carried out to adjust the resistivities of the model while keeping the boundaries fixed.

- After each iteration the theoretical curve is re-calculated and compared with the field data. This process is repeated until the RMS difference between the two curves reaches a minimum.
RESIST SOFTWARE

- RESIST is a processing package on PC computers for the processing and interpretation of resistivity data.
- It offers the user 3 options: Wenner, Schlumberger or Dipole-dipole arrays.

- The software package has a main instruction menu to control all operations, to make the package user-friendly and flexible in use.

- The package allows the user to manipulate the raw data: filtering high and low frequency noise (smoothing). This technique includes correction of single points and vertical curve-segment shift.

- All these manipulations are carried fully interactively on screen, so that the user has full control over these corrections.

- The package together with Geosoft mapping system software allows the user to prepare apparent resistivity cross-sections and apparent resistivity contour maps for a qualitative interpretation of the field observations.

- The quantitative interpretation techniques – interactive and batch processing – are included in the program structure. The quantitative interpretation of layer parameters is carried out using the indirect method.

How to use RESIST program

- Everything about how to use the program will be discussed in the lecture and lab.
Applications and Case Histories

1- Groundwater studies:

- The vertical electrical soundings provided a mean of obtaining information about the vertical distribution of fresh, brackish and saline water bodies and their areal extent.

- The vertical electrical soundings provided a means of obtaining information about the vertical distribution of fresh, brackish and saline water bodies and their areal extent.

- Electrical geophysics can help to identify the groundwater potential in areas and to assist in the planning of drilling programmes.

- Use of electrical resistivity method to identify the groundwater potential in areas led to decrease the borehole failure rate.

- Electrical methods are unique in giving information concerning the depth of the fresh water interface.

- Thick clay layer separating two aquifers can be detected easily on a sounding curve.

2- Mapping buried stream channels

- Horizontal profiling, electrical sounding or both are used to map buried stream channels accurately.

- Horizontal profiling can give information on the presence or absence of shallow buried stream channels.
Fig. 68: Apparent resistivity profile and geologic interpretation over buried channel

- Electrical sounding should precede and follow the horizontal profiling for the determination of depth.

3- Geothermal Studies

- Electrical sounding and horizontal profiling can be used to delineate a fault zone where stream can be tapped for energy.

- In the following figure, the two low resistivity areas outlined by the 5 ohmm contour are believed to delineate the hottest ground.
Fig 69: Map of apparent resistivity in geothermal area.

4- Mapping fresh – salt water interface.

- Fresh – salt water interface can be mapped successfully with resistivity sounding and horizontal profiling.

5- Mapping the water table

- The determination of the depth to the water table on a sounding curve is generally a more difficult problem.

- Under favorable conditions the water table can be detected on a sounding curve as a conductive layer.
6- Mapping clay layers

![Schlumberger sounding curves](image)

**Fig 70:** Examples of Schlumberger sounding curves show homogeneous sediments underlain by high resistivity Pre-Cambrian rocks (VES 26). VES 7 shows the presence of a thick section of low resistivity clay

- The clay layer can be detected on a sounding curve as a low resistivity layer where the lower aquifer acts as an electrical basement.

7- Landfills

- High resolution resistivity surveys can be used in investigation of closed landfills, particularly with respect to potential leachate migration.

- Both resistivity sounding and sub-surface imaging have been used very successfully.

- There is no such thing as a typical landfill some are extremely conductive, others are resistive to the surrounding media.
8- Engineering site investigation

- Electrical resistivity method could be used successfully to detect sub-surface collapse features such as cavities or caves in limestone.

9- Location of buried foundation such as metal chain – link fence and an old diesel tank.
References

------------------: Environmental geophysics
Practical Course

الأدوات
- كراسة رسم بساني ١٠ ورقية
- مسطرة + أستيكة + قلم رصاص سنون ٢/١ مم
- آلة حاسبة

1- Field exercises:

a- Resistivity profiles with Wenner & Schlumberger
b- Resistivity mapping with Rectangle method
c- Vertical Electrical Sounding with Schlumberger

2- Lab exercises:

a- Interpretation of two and three-layer VES curves of different types by curve matching (i.e. the VES curves are digitized from the master curves).
b- Interpretation of three-type curves of different types by two-layer master curves and auxiliary curves (i.e. the VES curves are digitized from the master curves).
c- Interpretation of four-layer curves by two-layer master curves and auxiliary curves.
d- Interpretation of VES curves by computer programs.
f- In a,b,c & d represent the results in the form of:
   1- Section of apparent resistivity
   2- Apparent resistivity map.
   3- Geoelectric sections

g- Calculation of Dar Zarrouk parameters from the VES curves.
h- Interpretation of some digitized VES curves by asymptotes